

UNIVERSIDAD MAYOR DE SAN SIMON
FACULTAD DE CIENCIAS ECONOMICAS
INGENIERIA FINANCIERA



TAREA DE CALCULO

PRACTICA N.º 5

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Práctica #5

DERIVADAS

Por definición, hallar las derivadas de las siguientes funciones:

1) $f = x^2 - 7x + 4$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{(x+\Delta x)^2 - 7(x+\Delta x) + 4 - (x^2 - 7x + 4)}{\Delta x}$$

$$\lim_{\Delta x \rightarrow 0} = \frac{x^2 + 2x\Delta x + \Delta x^2 - 7x - 7\Delta x + 4 - x^2 + 7x - 4}{\Delta x}$$

$$\lim_{\Delta x \rightarrow 0} = \frac{2x\Delta x + \Delta x^2 - 7\Delta x}{\Delta x} = \Delta x(2x + \Delta x - 7)$$

$$f'(x) = \lim_{\Delta x \rightarrow 0} = 2x + 0 - 7 = \boxed{f'(x) = 2x - 7}$$

2) $f = 2x^3 + 1$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{2(x+\Delta x)^3 + 1 - (2x^3 + 1)}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{2(x^3 + 3x^2\Delta x + 3x\Delta x^2 + \Delta x^3) + 1 - 2x^3 - 1}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{2x^3 + 6x^2\Delta x + 6x\Delta x^2 + 2\Delta x^3 - 2x^3}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{\Delta x(6x^2 + 6x\Delta x + 2\Delta x)}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \rightarrow 0} 6x^2 + 6x(0) + 2(0) = \boxed{f'(x) = 6x^2}$$

3) $f = \frac{1}{x+2}$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{\frac{1}{(x+\Delta x)+2} - \frac{1}{x+2}}{\Delta x}$$

$$\lim_{\Delta x \rightarrow 0} \frac{\frac{1}{(x+\Delta x)+2} - \frac{1}{x+2}}{\Delta x} = \frac{\frac{x+2}{(x+\Delta x+2)(x+2)} - \frac{x+\Delta x+2}{(x+\Delta x+2)(x+2)}}{\Delta x}$$

$$\lim_{\Delta x \rightarrow 0} \frac{\frac{-\Delta x}{(x+\Delta x+2)(x+2)}}{\Delta x} = \frac{-\Delta x}{\Delta x(x+\Delta x+2)(x+2)}$$

$$\lim_{\Delta x \rightarrow 0} \frac{-1}{(x+\Delta x+2)(x+2)} = \frac{-1}{(x+0+2)(x+2)} = \frac{-1}{(x+2)^2}$$

$$\boxed{f'(x) = \frac{-1}{(x+2)^2}}$$

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4) $f = \sqrt{x+7}$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{\sqrt{(x+\Delta x)+7} - \sqrt{x+7}}{\Delta x}$$

$$\lim_{\Delta x \rightarrow 0} \frac{\sqrt{(x+\Delta x)+7} - \sqrt{x+7}}{\Delta x} \cdot \frac{\sqrt{(x+\Delta x)+7} + \sqrt{x+7}}{\sqrt{(x+\Delta x)+7} + \sqrt{x+7}}$$

$$\lim_{\Delta x \rightarrow 0} \frac{(\sqrt{(x+\Delta x)+7})^2 - (\sqrt{x+7})^2}{\Delta x (\sqrt{(x+\Delta x)+7} + \sqrt{x+7})} = \frac{x+\Delta x+7 - x-7}{\Delta x (\sqrt{(x+\Delta x)+7} + \sqrt{x+7})}$$

$$\lim_{\Delta x \rightarrow 0} \frac{\Delta x}{\Delta x (\sqrt{(x+\Delta x)+7} + \sqrt{x+7})} = \frac{1}{\sqrt{x+7} + \sqrt{x+7}}$$

$$\lim_{\Delta x \rightarrow 0} \frac{1}{\sqrt{x+7} + \sqrt{x+7}} = \frac{1}{\sqrt{x+7} + \sqrt{x+7}} = f'(x) = \frac{1}{2\sqrt{x+7}}$$

5) $f = 3^x$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{3^{(x+\Delta x)} - 3^x}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{3^x (3^{\Delta x} - 1)}{\Delta x} = 3^x \cdot \frac{3^{\Delta x} - 1}{\Delta x}$$

$$3^x \cdot \lim_{\Delta x \rightarrow 0} \frac{3^{\Delta x} - 1}{\Delta x} = \ln(3) \Rightarrow f'(x) = 3^x \cdot \ln(3)$$

6) $f = \tan x = \frac{\sin x}{\cos x}$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{\tan(x+\Delta x) - \tan x}{\Delta x} \rightarrow \tan(a) - \tan(b) = \frac{\sin(a-b)}{\cos a \cdot \cos b}$$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{\frac{\sin(x+\Delta x) - \sin x}{\cos(x+\Delta x) \cos x}}{\Delta x} = \frac{\sin \Delta x}{\cos(x+\Delta x) \cos x} \cdot \frac{1}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{\sin \Delta x}{\Delta x} \cdot \frac{1}{\cos(x+\Delta x) \cos x} = 1 \cdot \frac{1}{\cos(x+0) \cos x}$$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{1}{\cos x \cos x} = \frac{1}{\cos^2 x} = f'(x) = \sec^2 x$$

Definir si las siguientes funciones son continuas en $x=0$

7) $y = x^3 + 3x + 3$

1. $f(0) = 0^3 + 3(0) + 3 = 3$ existe

2. $\lim_{x \rightarrow 0} x^3 + 3x + 3 = 0^3 + 3(0) + 3 = 3$

$$\lim_{x \rightarrow 0^+} x^3 + 3x + 3 = \lim_{x \rightarrow 0^+} x^3 + 3x + 3$$

$$0^3 + 3(0) + 3 = 0^3 + 3(0) + 3$$

$$0+0+3=0+0+3$$

$$\boxed{3=3} \text{ existe}$$

$$3. x^3+3x+3=f(0)$$

$$\boxed{3=3}$$

La función $y=x^3+3x+3$ es continua en $x=0$

$$8) f=\frac{3}{\sqrt{x}}$$

$$x=0$$

$$1. f(0)=\frac{3}{\sqrt{0}}=\frac{3}{0}=\infty \text{ no está definido}$$

$$2. \lim_{x \rightarrow 0} \frac{3}{\sqrt{x}} = \frac{3}{\sqrt{0}} = \frac{3}{0} = \infty$$

$$\lim_{x \rightarrow 0^+} \frac{3}{\sqrt{x}} = \lim_{x \rightarrow 0^-} \frac{3}{\sqrt{x}}$$

$$\lim_{x \rightarrow 0^+} \frac{3}{\sqrt{0^+}} = \lim_{x \rightarrow 0^-} \frac{3}{\sqrt{0^-}}$$

$$\lim_{x \rightarrow 0^+} \frac{3}{0^+} = \lim_{x \rightarrow 0^-} \frac{3}{0^-}$$

$$\boxed{+\infty = -\infty} \text{ no existe } \neq$$

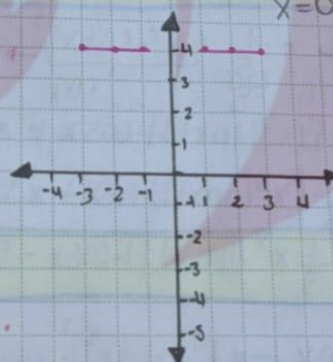
$$3. \lim_{x \rightarrow 0} \frac{3}{\sqrt{x}} = f(0) \quad \text{La función } f(x)=\frac{3}{\sqrt{x}} \text{ no es continua en } x=0$$

Determinar si la siguiente función es derivable

$$7) f=\frac{4x}{|x|}$$

$$x \neq 0$$

x	y
-3	-4
-2	-2
-1	-1
0	0
1	1
2	2
3	4



$$1. f(0)=\frac{4(0)}{|0|}=\frac{0}{0} \text{ NO.}$$

$$2. \lim_{x \rightarrow 0} \frac{4x}{|x|} = \frac{4(0)}{|0|} = \frac{0}{0} \text{ NO.}$$

$$\lim_{x \rightarrow 0^+} \frac{4x}{|x|} = \lim_{x \rightarrow 0^+} \frac{4x}{x}$$

$$\lim_{x \rightarrow 0^+} \frac{4x}{x} = \lim_{x \rightarrow 0^+} 4$$

$$\lim_{x \rightarrow 0^+} 4 = 4$$

$$\lim_{x \rightarrow 0^-} \frac{4x}{|x|} = \lim_{x \rightarrow 0^-} \frac{4x}{-x}$$

$$\lim_{x \rightarrow 0^-} \frac{4x}{-x} = \lim_{x \rightarrow 0^-} -4$$

$$\lim_{x \rightarrow 0^-} -4 = -4$$

$$\boxed{4 \neq -4} \text{ no existe } \neq$$

$$3. \lim_{x \rightarrow 0} \frac{4x}{|x|} = f(0) \quad \neq = \neq$$

La función $f(x)=\frac{4x}{|x|}$ no es derivable porque no cumple con las 3 características.

Derivar las siguientes funciones:

10) $f = 3^x + x^3 + 3x + 3$ $f'(x) = 3^x \cdot \ln 3 + 3x^2 + 3$

11) $f = \sqrt{x} + \frac{1}{\sqrt{x}} = x^{1/2} + x^{-1/2} \rightarrow f'(x) = \frac{1}{2}x^{-1/2} + (-\frac{1}{2})x^{-3/2}$
 $f'(x) = \frac{1}{2}x^{-1/2} - \frac{1}{2}x^{-3/2} = f'(x) = \frac{1}{2\sqrt{x}} - \frac{1}{2\sqrt{x^3}}$

12) $f = 2\ln x + x\ln 2 = 2(\ln x) + \ln 2(x)$
 $f'(x) = 2 \cdot \frac{1}{x} + \ln 2 = f'(x) = \frac{2}{x} + \ln 2$

13) $f = 5\cos x + x \cos 5 = 5(\cos x) + \cos 5(x)$
 $f'(x) = -5\sin x + \cos 5$

14) $f = x^3 \cdot \sin x \cdot e^{5x}$
 $f'(x) = (x^3)' \cdot \sin x \cdot e^{5x} + x^3 \cdot (\sin x)' \cdot e^{5x} + x^3 \cdot \sin x \cdot (e^{5x})'$
 $f'(x) = 3x^2 \cdot \sin x \cdot e^{5x} + x^3 \cdot \cos x \cdot e^{5x} + x^3 \cdot \sin x \cdot (e^5)^x \ln e^5$
 $f'(x) = (3x^2 \cdot e^{5x} + 5x^3 \cdot e^{5x}) \sin x + x^3 \cdot e^{5x} \cdot \cos x$
 $f'(x) = e^{5x} [(3x^2 + 5x^3) \sin x + x^3 \cdot \cos x]$

15) $f = \frac{x^x \cdot \cos x}{5} = \frac{1}{5} (x^x \cdot \cos x)$ $\rightarrow u \cdot v = u' \cdot v + u \cdot v'$
 $f'(x) = \frac{1}{5} [(x^x (\ln x + 1) \cdot \cos x + x^x \cdot (-\sin x))]$
 $f'(x) = \frac{1}{5} [x^x (\ln x + 1) \cos x - x^x \sin x]$
 $f'(x) = \frac{1}{5} x^x [(\ln x + 1) \cos x - \sin x]$

16) $f = x^8 \cdot \sin x \cdot \ln x \rightarrow f'(x) = (x^8)' \cdot \sin x \cdot \ln x + x^8 \cdot (\sin x)' \cdot \ln x + x^8 \cdot \sin x \cdot (\ln x)'$
 $f'(x) = 8x^7 \cdot \sin x \cdot \ln x + x^8 \cdot \cos x \cdot \ln x + x^8 \cdot \sin x \cdot \frac{1}{x}$
 $f'(x) = 8x^7 \cdot \sin x \cdot \ln x + x^8 \cdot \cos x \cdot \ln x + x^7 \cdot \sin x$
 $f'(x) = x^7 [(8 \cdot \sin x \ln x + x \cdot \cos x \ln x + \sin x)]$

17) $f = \frac{5^{2x} \cdot \cos x \tan x}{x^6 \cdot 6^{2x}} = \frac{5^{2x} \cdot \tan x}{x^6 \cdot 6^{2x}}$

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$$f'(x) = ((5^x)^x \tan x)' (x^6 2^x) + 5^x \tan x (x^6 2^x)'$$

$$(x^6 \cdot 2^x)^2$$

$$f'(x) = 6 \cdot 5^x \ln 5 \tan x + 5^x \sec^2 x (x^6 2^x)' - (5^x \tan x)' (6x^5 2^x + x^6 2^x \ln 2)$$

$$f'(x) = 25^x ((\ln 25) \tan x + \sec^2 x) (x^6 2^x)' - (25^x \tan x)' 36^x (6x^5 + x^6 \ln 36)$$

$$(x^6 \cdot 2^x)^2$$

$$18) f = \frac{(2^x - 1)(2^x + 1)}{(4^x + 1)(4^x - 1)} = \frac{(2^x - 1)(2^x + 1)}{(4^x + 1)(2^x - 1)(2^x + 1)} = f = \frac{1}{4^x + 1}$$

$$f'(x) = \frac{(1)' - ((4^x)' \cdot 1 + 4^x \cdot 0)}{(4^x + 1)^2} = \frac{0 - (4^x \ln 4 + 0)}{(4^x + 1)^2}$$

$$f'(x) = \frac{-4^x \ln 4}{(4^x + 1)^2}$$

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